ADAPTIVE NON-UNIFORM RATE SAMPLING AND APPLICATION IN DATA COMPRESSION

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Abstract - In this paper the author considers a general method, based on time domain samples for spectral manipulation of time limited signals. First, the original signal is divided into some frames in the time domain. Then, by presenting a suitable theoretical and computational algorithm, and using a method for improving the speed of convergence, we find the local bandwidth of each frame; thereby, each frame is sampled with a new rate, so the amount of data per frame is decreased. Finally, by storing suitable information at the beginning of each frame, the original signal could be reconstructed. Therefore, by this method some redundant data is removed and the net volume of data is reduced. The reconstruction of original signal is achieved by using periodic interpolation. In this technique, a method for reduction of error (due to false interpolation) at the end of any frame is examined carefully.

Keywords - Data Compression, Local Bandwidth Estimation, Interpolation, Time Domain Sampling, Adaptive Non-uniform Rate Sampling, Spectral Analysis.

INTRODUCTION

Nowadays, construction and improvement of powerful digital processors have given impetus to work and research in the fields of data, speech and image processing. In order to use and apply such computational ability, it is necessary to convert original analog signals, to be processed, into digital waveform through the sampling process.

Evidently, more efficient sampling process results in greater ability in digital signal processing procedures. Reducing the number of digital samples, as well as solving the problem of limited capacity of communication channel [2] are the other advantages of efficient sampling.

We generally use the Nyquist criterion to determine the sampling rate. This criterion computes the sampling frequency based on the maximum frequency of the given analog signal, and we could reconstruct band limited signals by sampling values with the use of Shannon Interpolation series [2,9]. For the band pass signals, the modified Nyquist criterion which is based on the differences between the maximum and minimum frequencies is applied [8,9]. The exact interpolation of such signals is achieved by using the band pass interpolation series [10,11].

In using Nyquist criterion and Shannon series, we consider the sampling process with constant, uniform spaces between adjacent sampling instants, but in some applications – i.e. machine vision, radio astronomy [3], Tomography [13], motion compensation of time varying imagery, geology [14], etc. – we encounter some non-uniformly spaced samples. In these cases, for reconstructing original waveform some powerful theorems with mathematical foundations have been planned and proved [1,5].

ADAPTIVE SAMPLING

Assume that the least information for sampling a special analog signal, that is the maximum frequency, and the number of bits for desired precision are given. Now, we do sampling process, with the sampling frequency at least twice the maximum frequency. For example, consider (Figure 1) and divide this waveform into three parts in time domain (A,B,C). A simple and basic definition for frequency states that it is the number of oscillations in unit time. So, the instantaneous frequency of the middle part (B) is higher than the other parts (A,C), and from a shaping standpoint for drawing (reconstructing) this part, we require a larger number of samples. But, in the first part (A) it is a dc level, and one sample is required, and in the third section (C) we need a lesser number of samples. In other words, in parts (AC) there is redundant data that is excessive for reconstruction, and could be removed without causing any problem for reconstructing the signal.

If we take the minimum number of samples in each frame, using the above method (Adaptive Sampling), there will be a set of irregularly spaced samples. Therefore, in usual form, we couldn't reconstruct the original signals, but if non-uniform sampling theorems were considered, we could expect the reconstruction of the original signal with tiny errors. There are two important points:

I- According to the above procedure, we descriptively designated to each frame one maximum frequency. In the next section, we attempt to provide suitable theoretical and computational definitions for computing these frequencies.

II- If we use non-uniform sampling, we will require the information about the instance of sampling, which is in contradiction to data compression. For this reason, in the next section this problem is solved.

EFFECTIVE MAXIMUM FREQUENCY

We need a set of data for defining the effective maximum frequency for a determined duration of an arbitrary signal X(t). If we divide the signal X(t) into the non-overlapped frames, and for each frame determine a unique frequency, which is inserted at the beginning of the data in that frame, we will practically overcome the second problem that was mentioned in the pervious section.

– PERIODIC SPREADING OF A TIME LIMITED SIGNAL

Consider g(t) as a part of arbitrary signal X(t), which is confined to the time interval I=[a,b] and is zero elsewhere as shown in Figure 2. First, we consider h(t) as a periodic function, and g(t) is one period of h(t). Although we have some theoretical problems in defining the limited maximum frequency for g(t) due to the effect of reciprocal spreading [2], this problem is diminished for h (t).

– COMPUTATIONAL DEFINITIONS

We must consider an error criterion to define a practical and computational definition for the maximum frequency. We use the percent root mean square error (PRD) [7] criterion which is defined as follows:

$$PRD\ (p,q) = 100 * \sqrt{\sum_{i=1}^{n} [P\ (i) - q(i)]^2 / \sum_{i=1}^{n} p(i)^2} \qquad (1)$$

Where the vector P is the original data and vector P represents the reconstructed data. In fact, this form is similar to the mean square error (MSE) criterion, which is normalized by the signal power.

As mentioned before, g(t) is defined in the time interval I= (a,b) with duration (d=b-a) and (n) is the original number of samples of g(t) with sampling period (Ts) and associated maximum frequency (f_m=2/Ts). So, the relation between (f_m,d,n) is $\mathbf{n=2df_m}$. Therefore, the numbers of samples in this duration determine maximum frequency.

Now, we consider the following notations for defining the effective maximum frequency in terms of number of samples. We define the following parameters:

- E: Allowable error in terms of PRD
- z(*l*): It contains the **l** equidistance samples' value in the interval **I**, computed by the (n) original data in I.
- $g_{z(l)}$: Denotes the reconstructed **g** in the (n) original points by z(*l*).

We define (*l*) as the minimum possible number of samples with allowable error (E) in the interval I, when it satisfies the following inequality:

$$PRD\ (g, g_{Z(l)}) < E < PRD\ (g, g_{z(l\_1)}) \qquad 2 < l \le n \qquad (2)$$

We choose the value for z by the mean value of data when (*l*=1). Then, we compare the resulting PRD with E, if the error is less than allowable error we take (*l*=1); otherwise, we inspect the relation (2).

In the above method for each change in *(l)* and for comparison with the resulting error, we need to interpolate two functions. In practice, even with a high speed processor, this direct method is so slow that some authors abandon it totally approach [4], but we perceive that the above method could give very good response in the time domain error, so we wouldn't ignore this method and would attempt to improve the speed of convergence.

– THE PRD CURVE

To describe the behavior of PRD curve, we consider the arbitrary signal X(t), and take g(t) (Figure 2) as a time limited part of X(t), with (n) original samples' value, and define the new parameter ($k = n - l$, $1 < l \leq n$). The general form of PRD curve versus the variable (k) is given in (Figure 3). The curve generally increases, except for (l=1), because the approximation of function with mean value of samples, could decrease the PRD.

THE DIFFICULTIES OF INTERPOLATION

As said before, we consider **(h)** as a periodic version of **(g)**, and work on it. Although there exist other forms of interpolation such as spline method [15], the results show that spline fails specially at the end of time limited duration (Figure 4). So, it is not suitable for our work. Use of normal form of Shannon's series interpolation, diminishes and fails at the end of each time limited interval; although, the fitness is relatively good (Figure 5). This behavior could be explained by considering the Shannon's formula. It considers all the samples out of the interpolation duration as zeros, and the series tries to reach these zero values, and so they are always diminished at the end.

In the method where ***(h)*** is used for interpolation **(g)**, it is important to note that **(g)** is the time limited part of an arbitrary signal, so there is no reason for equality of g(a)=g(b) (Figure 2). For this reason, the interpolation of (g) while considering it a part of (h) is failed at the end; because, the interpolation of *(h)* produces smooth continuous function that aims at reaching the g(a) at the end of interval I=[ab) (Figure 6). Therefore, response of periodic interpolation at the end of each frame will depend on the equality between g(a) and g(b), and this subject produces signal dependent method, that is in contradiction to our aim, which is signal independency of the method.

We solve the problem of interpolation in using (h) with the following procedure:

We assume that the (n) original samples of (g) are located at t, and when the number of samples become (1), the new points are distributed in s:

$$t_k = a + (k - 1)T_s \qquad (k1,2\ldots,, n) \qquad (3)$$
$$s_m = a + (m - 1)T_{s2} \qquad (m= 1, 2,..., 1)$$

Practically, deviation of reconstructed signal is occurred in the interval (s_l,t_n). The final choice for removing this problem is achieved by storing the g(t_n), and then we reconstruct the signal between s and t, by special interpolation. Here, we use the linear interpolation (Figure 7), although with considering the other marginal information (produced by the interpolation of h), we can apply polynomial interpolation with higher order, but the results show these approximations are weaker than liner interpolation (Figure 8). In addition, it is possible for a special (g), the order two or three, to have a better response (Figure 9). But in this case, the linear interpolation has a near error response to the other order, and we generally guarantee the linear form to any (g), but this guaranty doesn't exist for higher orders (Figures 8-10, and Table 1).

Table 1: Specification of Figures 8-10.

Figure	Reconstruct the end of Figure	n, l	Periodic Dash Dash	Linear Solid	order2 Dot Dot	order3 Dash Dot	Original Signal Solid
8	4	191,19	8.71	3.46	3.68	5.37	Prd:
9	5	229,23	8.13	8.81	8.13	8.11	Prd:
10	6	191,15	26.27	9.31	11.3	14.74	Prd:

IMPROVING THE SPEED OF CONVERGENC`E

To determine (*l*) within equation 2, we must find the nearest integer *(l)* in crossing the PRD curve with horizontal threshold E (Figure 3). In other words, we want to find the "zero" of nonlinear function ($f(k) = PRD(k) - E$). Some classical methods exist for finding zeroes of continuous nonlinear function [6] – for example: 1- Bisection, 2- False Position, 3- Newton Raphson Slope Search, 4- Secant .... Direct applying of these methods on the discrete function f(k), produces problems of divergence, oscillation and instability. We modified the above methods by suitable programming and inspecting the methods, 1,2,4, as well as by combining the method 2 with 4. At the end, to gain a better response we consider the new method, that is described in the next section.

– A NEW METHOD TO BISECT INTERVAL

We consider two parametric nonlinear functions. If the coordinates of marginal points are (i, f_i) and (j, f_j), among the tested two parametric nonlinear functions, we could denote the suitable form by $\mathbf{f - f_i = A(k - i)^S}$ **(i<j).** In this relation, the proper choice for (s) is between (2-2.5), and preferably 2. The parameter A is computed by applying the other point.

– THE COMBINATION OF TIME AND FREQUENCY COMPUTATION

As said before, our aim was data compression by applying a time domain error. To speed up the convergence of algorithm, we could use Fourier transform of (g), and with combining time and frequency domains processing, not only reach our goal, but also improve the speed of the method. To get to this objective, it is better to choose (n), to be the power of two to use the advantages of the FFT computations. Briefly, in this method the magnitude of Fourier transform of (g) without considering the dc level of (g) is computed. Then, we apply the variable horizontal threshold level on this magnitude, and the concept of frequency domain PRD, with the basis of allowable frequency domain error inspect. With this procedure, the effective maximum frequency is found and the resampling is done. The number of data in the resampling process is near to that found with the time domain method. Then, the algorithm is followed in time domain, until the final result is achieved. As a simple example, consider (Figure 11). The result is collected in (Table 2) and (Figure 12). In the computation of frequency magnitude, after removing the dc level of each frame, the result is multiplied by the Blackman window [12], with the following formula, and then the frequency magnitude is computed, and processed.

$$W(k) =.42-.5\cos(2\pi.\ k / n) +.08\cos(4\pi.\ k / n)\ (0 < k < n)\ \text{(Black -man Window)} \qquad (4)$$

According to the PRD criterion in the time and frequency domain, allowable errors in each frame are taken 7 and 20, respectively. The original signal (Figure 11) is divided into ten segments (I =1,2,...10), and the original number of samples in each frame is (n=128). The total error between the original and reconstructed signals (Figures 11-12) is *PRD=3.39, (SN=-29.4* dB), and the compression ratio is about 6.2.

Table 2: Specifications of Figures 11-12.

I	1	2	3	4	5	6	7	8	9	10
l	11	9	1	105	41	7	3	1	2	4
prd	5.6	3.4	0.71	1. 8	0.66	6.4	1.9	0	2.5	5.1

CONCLUSIONS

In this paper, we presented an algorithm for adaptive non-uniform rate sampling, with suitable theoretical and computational definitions. In practice, we encountered problems of incorrect interpolation specially at the end of each time limited frame (Figures 4-6). To solve these problems, we illustrated with suitable figures and numerical values, that by storing the last sample of each frame as a key-data and applying the linear interpolation (Figures 7-10 and Table 1), we could properly interpolate time limited signals. Therefore, this method preserves the shape of signal especially in the most critical regions, i.e. at the end of each frame (Figure 12). Besides, it is a suitable method which removes and reduces the redundant data with an acceptable error (Figures 11-12 and Table 2).

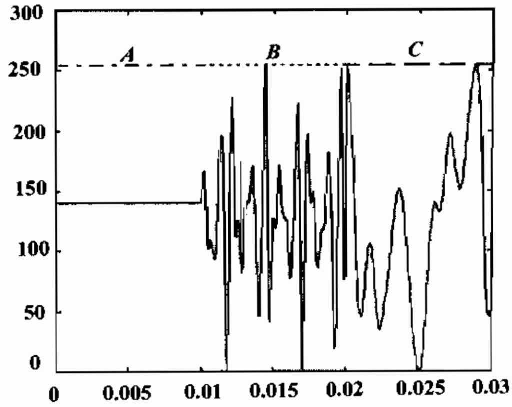

Figure 1: x(t).

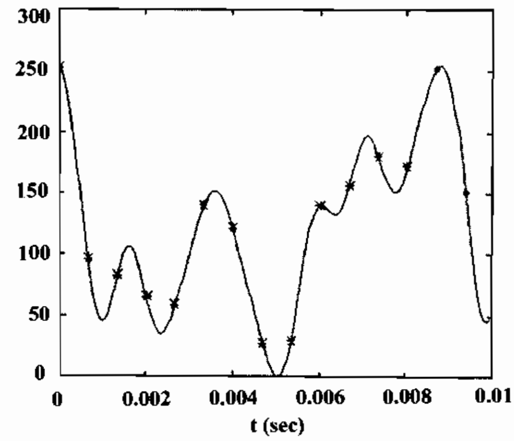

Figure 2: g(t), and it's resampling: (n,1)=(191,15).

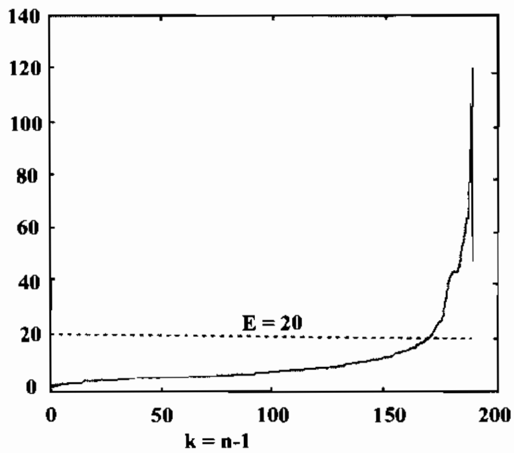

Figure 3: PRD curve.

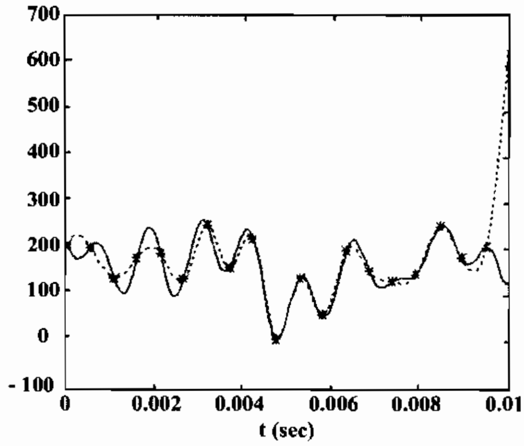

Figure 4: Dot, Spline (prd =40.63) (n,1)=(191,19).

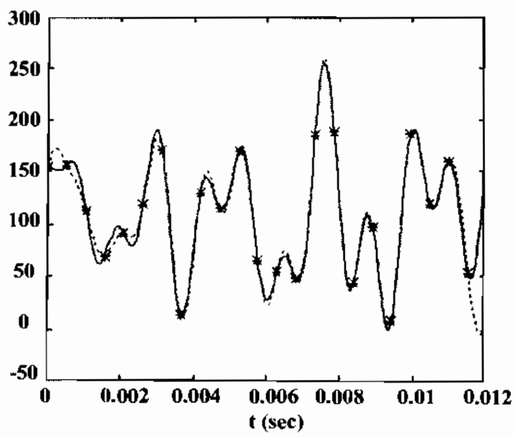

Figure 5: Dot, Shannon (prd=15.43) (n,1)=(229,23).

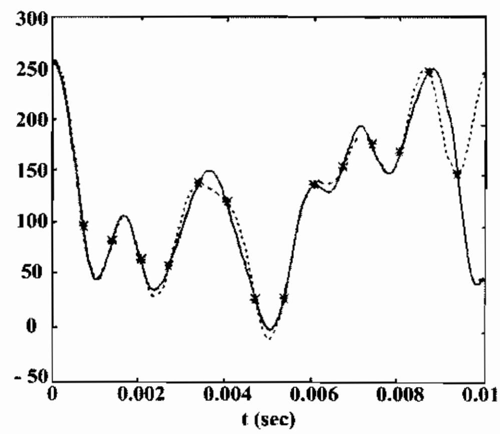

Figure 6: Dot, Periodic (prd=26.43) (n,1)=(191,15).

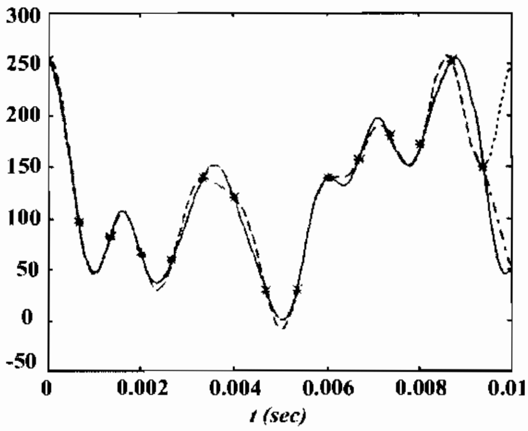

Figure 7: Dash Dot, removing the end effect of Figure 6.

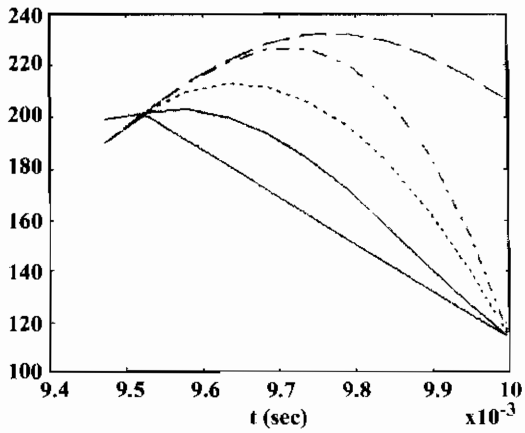

Figure 8: Refer to Table 1.

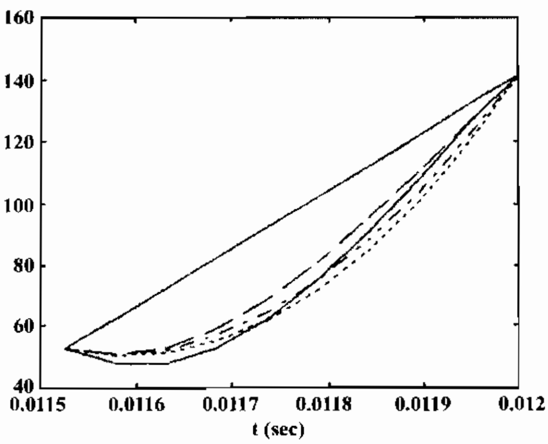

Figure 9: Refer to Table 1.

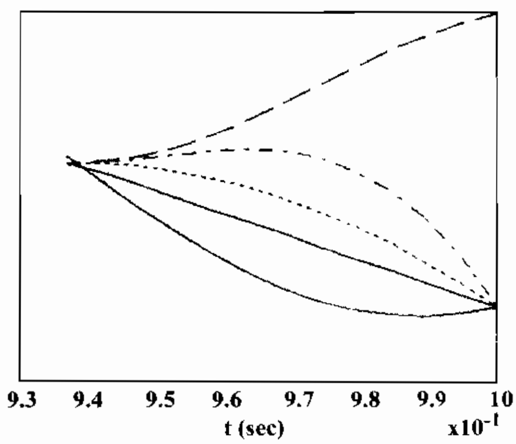

Figure 10: Refer to Table 1.

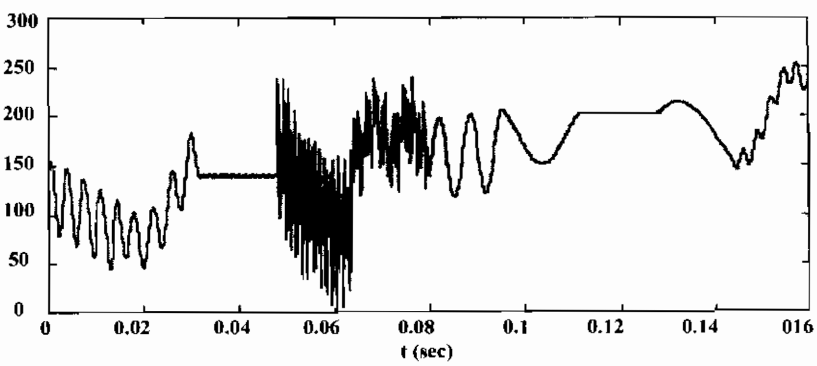

Figure 11

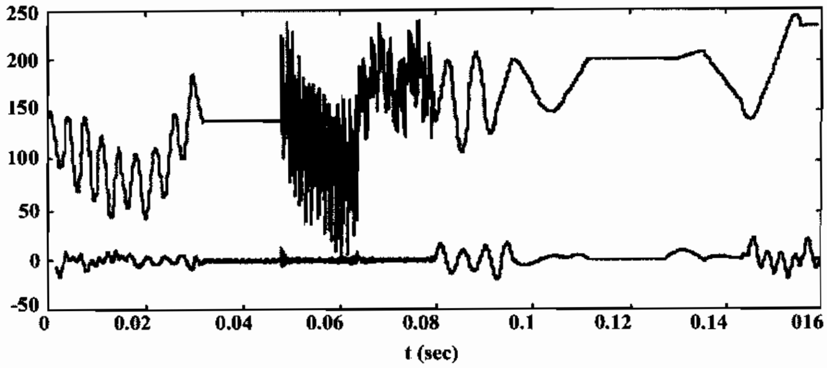

Figure 12: The reconstruction of Figure 11 and the error signal. Refer to Table 2

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