APPLICATION OF PASSIVE GG-AERO STABILIZATION FOR NEAR-EARTH SMALL SATELLITES

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Abstract - When the orbital altitude of a small satellite is near-Earth, passive stabilization can be considered. The purpose of this paper is to combine the passive gravity gradient and aerodynamic stabilization methods together so that all the three axes stabilizations can be obtained. In this regard, the set of nonlinear equations of motion will be solved numerically with specified initial condition. The history of the attitude stabilization will be investigated and discussed in detail. The paper demonstrates the feasibility of passive gg-aero stabilization and provides guidelines for the satellite design to obtain the best attitude performance without sacrificing satellite lifetime.

Keywords - Gravity Gradient(GG) Torque, Aerodynamic(Aero) Torque, Small Satellite, Low Earth Orbit, Passive GG-Aero Stabilization, Attitude Control, GG Boom, Vertical Tail Stabilizer.

INTRODUCTION

Satellites are continuously subjected to perturbation torques while in orbit. On board torques may be caused by the motion of mechanisms, reaction or momentum wheels and the firing of thrusters. External torques are produced by the Earth's gravity gradient, interaction with the Earth's magnetic field, solar radiation pressure, thermally induced flexure in satellite structures, and aerodynamic drag in near Earth. It is the job of the attitude determination and control subsystem (ADCS) to measure and counter these perturbations to ensure that the spacecraft can perform its mission. These adjustments include maintaining orbital position and satellite pointing accuracy.

Several control methods have been developed over the past years since the first satellite was launched. Generally speaking, all those techniques may be classified into two groups: active or passive. Active techniques are required for missions where the availability of high pointing accuracy is vital. Passive stabilization of satellites, based on aerodynamic drag and gravity gradient torques, could be a desirable low cost, low power consumption, low weight control principle for low altitude missions in the future. That is why the use of passive stabilization is very frequent.

The problem that the stability of satellites may be influenced by gravity gradient and aerodynamic torques for low earth orbit (LEO), has been discussed in the literature. Wall gave an early discussion on the feasibility of aerodynamic attitude stabilization of a near-Earth satellite vehicle [11]. More eminent researches have been performed by Ravindran and Hughes [7]. A rather complete survey on the influence of aerodynamic forces on the attitude motion of the near-Earth satellite system has been presented by Shrivastava and Modi [9]. In practice, a successful example is the use of aerodynamics for the pitch control of COSMOS-149 [8]. Meirovith and Wallace have considered the effect of aerodynamics and gravitational torques on a rigid, symmetric satellite that is a body of revolution and is spinning [6]. More akin to the present problem, Garber has shown by linearized analysis, is that for a constant torque in the direction normal to the orbital plane, a symmetric, gravity-stabilized satellite will experience unstable motion.

Although, in general, the aerodynamic torque cannot be modeled so simply, the result does give some insight into the problem; because, in the linearized equations part of the aerodynamic torque contribution is a constant term [3]. Gerald presented details of the derivation of the aerodynamic torque for the general case. However, for the purpose of an analytical stability analysis, some restrictions on the shape of the satellite are imposed, but they do not detract seriously from the practicality of the problem. It is assumed that the body has a plane of geometrical symmetry and the center of mass lies in this plane. It is also assumed that the coordinate system - this system is denoted as the geometric axes - where the aerodynamic coefficients are computed is aligned detracting with the principal axes. This last assumption, although detracting from the flexibility of the equations leaves their generality intact [4]. Bak and Wisniewski showed how the instability could be avoided, and how the aerodynamic drag could be used for passive attitude stabilization of a satellite with an appropriate geometry. Design recommendations and tradeoffs are presented for the use of aerodynamic drag in attitude control [1]. In these references in order to find aerodynamic torque effect, satellite is modeled as a three orthogonal plane. Crosscoupling is minimized by symmetric design and optimization of performance index reduces the ballistic coefficient.

An important point to mention here is that increasing the center of mass-pressure offset reduces satellite lifetime. Aero-stabilization has previously been addressed in [5] for the shuttle Hitchhiker GAMES mission. The results focused on the effect of altitude decay and did not consider gravity gradient. The problem of stability of a rigid satellite that only influenced gravitational torques has been discussed in the literature [2,10,12]. Also, gg-stabilization has been studied extensively. The purpose of this paper is to design a novel GG and Aero passive stabilization method in such a way that the life time of the system is increased as the emergence of the inverted boom, that occurs by effect of aerodynamic disturbances, in the satellite is prevented. This is while no new constraint is added to the satellite. Aerodynamic drag torques are used to compensate for the gravity gradient effects that provide a passive stabilization of the satellite. Analyses of the attitude motion and dynamic equations of a satellite under the action of both gravity gradient and aerodynamic torques will also be presented. In this regard, gravity torque is produced by gravity gradient boom (long tube and tipmass). This configuration creates dumb bell properties with satellite body and improves moments of inertia distribution. It is most important to point out that because a gravity gradient system cannot generate torque along boom axis, we may add a vertical tail stabilizer perpendicular to the boom axis. Then, by designing a simple feedback differential control loop, yaw motion is stabilized. Finally, these two stabilization methods will be combined together so that the three-axes stabilization of satellite may be obtained analytically. Desired results are demonstrated by simulation of this method on nonlinear equations of a satellite motion.

Note: All parameters in this paper are described in Appendix A.

SATELLITE BASIC EQUATIONS

– EQUATIONS OF MOTION

Several basic coordinates are used in defining satellite attitude and position. The geometry of the keplerian orbit on a satellite with the center of mass o moving around the center of force O is shown in Figure 1. The $OXYZ$ is the inertial coordinate mounted in the center of earth, where OY is normal to the orbital plane and OZ passes the perigee. The $ox_oy_oz_o$ is the rotating coordinate with oy_o normal to the orbital plane and oz_o in direction of the local vertical. Therefore, OY and oy_o are parallel. The $oxyz$ represents the principal body coordinate system. Major and minor moments of inertia are oy and oz axes, respectively. In all the above-mentioned coordinates, the 3rd axis is determined so that the obtained coordinate system will be the right angle. If we rotate $OXYZ$ by angle η around the OY -axis and then translate the origin to point o, we will have the system $ox_oy_oz_o$. Then, by rotating a roll angle θ_1 around ox_o-axis, a pitch angle θ_2 around oy_1-axis, and a yaw angle θ_3 around oz-axis, the spatial orientation of the $oxyz$ system will be completely specified, as is shown in Figure 2.

The transformation matrix from the coordinate system $ox_oy_oz_o$ to the principal body coordinate system $oxyz$ is as follows [11]:

$$l = \begin{bmatrix} c_2c_3 & c_2s_3 & -s_2 \\ s_1s_2c_3 - c_1s_3 & c_1c_3 + s_1s_2s_3 & s_1c_2 \\ s_1s_3 + c_1s_2c_3 & c_1s_2s_3 - s_1c_3 & c_1c_2 \end{bmatrix} \tag{1}$$

where, $c_i \equiv \cos(\theta_i)$, $s_i \equiv \sin(\theta_i)$. The angular velocity vector of the satellite with respect to the inertial frame can be expressed as [11]:

$$\omega = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \dot\theta_1 - \dot\theta_3 s_2 + \omega_\circ c_2 s_3 \\ \dot\theta_2 c_1 - \dot\theta_3 c_2 s_1 + \omega_\circ (c_1 c_3 - s_1 s_2 s_3) \\ \dot\theta_3 c_1 c_2 - \dot\theta_2 s_1 + \omega_\circ (c_1 s_2 s_3 - s_1 c_3) \end{bmatrix} \tag{2}$$

In Eq. 2, ω_x, ω_y and ω_z are the angular velocity components around x,y,z body coordinate axes and $\omega_\circ$ is the orbital velocity of the satellite.

– DYNAMIC OF A RIGID BODY

The equation of rotational motion of a rigid body is [11]:

$$T = T_c + T_d + \dot H \tag{3}$$

Where T_c is the control torque and T_d is the disturbance torque, that affect the satellite. $\dot H$ is the angular momentum of the rigid body. The reference point for calculating T and H is the center of the mass of the rigid body or a fixed point. Now, if the $oxyz$ coordinate system is chosen such that the coordinate axes are principal axes at the center of the rigid body mass, then all products of inertia will vanish and Eq. 3 becomes:

$$\begin{aligned} T_x &= I_{xx}\,\dot\omega_x + (I_{zz} - I_{yy})\,\omega_y\,\omega_z \\ T_y &= I_{yy}\,\dot\omega_y + (I_{xx} - I_{zz})\,\omega_z\,\omega_x \\ T_z &= I_{zz}\,\dot\omega_z + (I_{yy} - I_{xx})\,\omega_x\,\omega_y \end{aligned} \tag{4}$$

These equations are known as the Euler's equations.

The differential equations for the three components of can be rewritten as:

$$\begin{aligned} \dot\omega_x &= \frac{T_x}{I_{xx}} - \frac{I_{zz} - I_{yy}}{I_{xx}}\,\omega_y\,\omega_z \\ \dot\omega_y &= \frac{T_y}{I_{yy}} - \frac{I_{xx} - I_{zz}}{I_{yy}}\,\omega_z\,\omega_x \\ \dot\omega_z &= \frac{T_z}{I_{zz}} - \frac{I_{yy} - I_{xx}}{I_{zz}}\,\omega_x\,\omega_y \end{aligned} \tag{5}$$

From Eq. 2 we can derive the equations for $\dot\theta_1, \dot\theta_2, \dot\theta_3$ as:

$$\begin{aligned} \dot\theta_1 &= \omega_x + \frac{s_2}{c_2}(\omega_y s_1 + \omega_x c_1) - \omega_\circ \frac{s_3}{c_2} \\ \dot\theta_2 &= \omega_y c_1 - \omega_z s_1 - \omega_\circ c_3 \\ \dot\theta_3 &= \frac{1}{c_2}(\omega_y s_1 + \omega_z c_1) - \omega_\circ \frac{s_2 s_3}{c_2} \end{aligned} \tag{6}$$

Thus, Eqs. 5 and 6 are the three-dimensional equations of motion for attitude motion of the satellite. Now, by inserting Eq. 2 in Eq. 4, satellite equations of motion will be obtained based on Euler's angles. To analyze small-angles, these equations can be linearized as follows:

$$\begin{aligned} T_x &= I_{xx}\ddot{\theta}_1 + \omega_\circ^2 (I_{yy} - I_{zz})\theta_1 - \omega_\circ (I_{xx} - I_{yy} + I_{zz})\dot{\theta}_3 \\ T_y &= I_{yy}\theta_2 \\ T_z &= I_{zz}\ddot{\theta}_3 + \omega_\circ^2 (I_{yy} - I_{xx})\theta_3 - \omega_\circ (I_{xx} - I_{yy} + I_{zz})\dot{\theta}_1 \end{aligned} \tag{7}$$

Since moments of inertia affect the stability, we define the following parameters:

$$\begin{aligned} \sigma_x &= \frac{I_{yy} - I_{zz}}{I_{xx}} = \frac{\int (x^2+z^2)\,dm - \int (x^2+y^2)\,dm}{\int (z^2+y^2)\,dm} = \frac{\int (z^2+y^2)\,dm\ 1}{\int (z^2+y^2)\,dm} < 1 \\ \sigma_y &= \frac{I_{xx} - I_{zz}}{I_{yy}} = \frac{\int (y^2+z^2)\,dm - \int (x^2+y^2)\,dm}{\int (z^2+x^2)\,dm} = \frac{\int (z^2+x^2)\,dm\ 1}{\int (z^2+x^2)\,dttm} < 1 \\ \sigma_z &= \frac{I_{yy} - I_{xx}}{I_{zz}} = \frac{\int (x^2+z^2)\,dm - \int (z^2+y^2)\,dm}{\int (z^2+y^2)\,dm} = \frac{\int (x^2+y^2)\,dm\ 1}{\int (x^2+y^2)\,dm} < 1 \end{aligned} \tag{8}$$

x, y, z are distances of dm from the satellite mass center in the three principal axes directions and is the mass of any satellite particle. In Eq. 8 we can write the second parameter with the use of two other parameters, i.e:

$$\sigma_y = \frac{\sigma_x - \sigma_z}{1 - \sigma_x \sigma_z} \tag{9}$$

By inserting the results of Eq. 8 in Eq. 7, satellite linearized equations of motion can be obtained as the following general equations:

$$\begin{aligned} T_x / I_{xx} &= \ddot{\theta}_1 + \omega_\circ^2 \sigma_x \theta_1 - \omega_\circ (1-\sigma_x)\dot{\theta}_1 \\ T_y / I_{yy} &= \ddot{\theta}_2 \\ T_z / I_{zz} &= \ddot{\theta}_3 + \omega_\circ^2 \sigma_z \theta_3 - \omega_\circ (1-\sigma_z)\dot{\theta}_1 \end{aligned} \tag{10}$$

The above equations will be used for stability analysis.

EXTERNAL TORQUES

The external torques acting on the satellite that need to be considered are the gravity gradient torque *(T_{gg})* and the aerodynamic torque *(T_{ad})* as shown in Figure 3:

$$T = T_{gg} + T_{ad} \tag{11}$$

All other torques are regarded as disturbances and will be neglected. These torques include, solar radiations torque and magnetic torque that are lower than 10^{-6} N.m.

– GRAVITY GRADIENT TORQUE

The gravity gradient torque acting on the satellite is [11]:

$$T_{gg} = \frac{3\mu}{R^3}\begin{bmatrix}(I_{zz} - I_{yy})\, sin\theta_1\, cos\theta_1\, cos^2\theta_2 \\ (I_{zz} - I_{xx})\, cos\theta_1\, cos\theta_2\, sin\,\theta_2 \\ (I_{xx} - I_{yy})\, sin\theta_1\, sin\theta_1\, cos\,\theta_2\end{bmatrix} \qquad (12)$$

Equilibrium stiffness, assuming that isotropic rods are mounted in any of the three axes, are defined as:

$$\begin{aligned} &roll\ stiffness \approx (I_{yy} - I_{zz}) = (I_{zzb} - I_{zzb}) + \frac{2}{3}(m_3 l_3^2 - m_2 l_2^2) \\ &pitch\ stiffness \approx (I_{xx} - I_{zz}) = (I_{xxb} - I_{zzb}) + \frac{2}{3}(m_3 l_3^2 - m_1 l_1^2) \\ &yaw\ stiffness \approx (I_{yy} - I_{xx}) = (I_{yyb} - I_{xxb}) + \frac{2}{3}(m_1 l_1^2 - m_2 l_2^2) \end{aligned} \qquad (13)$$

I_{xxb}, I_{yyb}, I_{zzb} are satellite principal moments of inertia about roll, pitch and yaw axes, respectively, m_i is *i* th tipmass, l_i is th length of rod and $i=1,2,3$ indicates satellite principal axes direction. In this paper, the under design satellite mission is nadir orientation. So, we should increase robustness of roll and pitch axes. To do so, we increase stiffness of roll and pitch axes by putting l_1 and l_2 equal to zero and l_3 possibly large in order to provide satellite attitude control system requirements and fulfill the mission. Thus, boom is mounted in the yaw direction, where yaw stiffness is very low. Therefore, when angles are small, gravity gradient torque can be obtained in the form of the following linearized equation:

$$Tgg = 3\omega_o^2 \begin{bmatrix}(I_{zz} - I_{yy})\,\theta_1 \\ (I_{zz} - I_{xx})\,\theta_2 \\ 0\end{bmatrix} \qquad (14)$$

As it is clear, no torque is generated along the yaw axis. Also, it should be noted that values of moment of inertia are too close when the satellite does not have a boom [2,4].

– AERODYNAMIC TORQUE

The interaction of the upper atmosphere with a satellite's surface, produces a torque about the center of the mass. The dominant environmental disturbance torque for a spacecraft is below 500km. Now, approximating the satellite structure by a collection of simple geometrical elements, the aerodynamic torque can be found as the vector sum of the torques on the individual elements composing the surface of the satellite. To simplify this expression, assume that the satellite is modeled as a number of plane surfaces. Then, forces and torques can be calculated per any element or set of elements, in which summation of them will express aerodynamic torque and force on the satellite as follows [1,10,12]:

$$T_{ad} = F\,(T_{pa} - C_g)$$
$$F = 0.5\,(\rho C_D A V_R^2) \qquad (15)$$

C_{pa} is the aerodynamic center of pressure and C_g is the atmospheric drag coefficient. A definition of other parameters appears in appendix A.

Satellite linearized equations of motion, with gravity gradient and aerodynamic torques, are obtained from combination of Eqs. 10, 11, 13 and 15 as follows:

$$\ddot{\theta}_1 + 4\omega_\circ^2\,\sigma_x\,\theta_1 - \omega_\circ(1-\sigma_x)\,\dot{\theta}_3 = 0$$
$$\ddot{\theta}_2 + 3\omega_\circ^2\,\sigma_y = 0 \qquad (16)$$
$$\ddot{\theta}_3 + \omega_\circ^2\,\sigma_z\,\theta_3 + \omega_\circ(1-\sigma_z)\,\dot{\theta}_1 = T_{ad}$$

In Eq. 16, gravity gradient torque influences roll and pitch axes, and aerodynamic torque has an effect on the yaw axis. Therefore, in low altitudes, where these torques are large, they affect all three axes of the satellite and cause instability.

In this paper, by addition of a gravity gradient boom in the yaw axis direction, gravity restoring torque can be provided. Moments of inertia will also be augmented in the plane perpendicular to yaw axis, so that roll and pitch axes will be stable.

It should be noted that since aerodynamic disturbance torque has an effect on the yaw axis, it may cause boom inverting condition. To overcome this problem, in this paper, a tail stabilizer was added, Figure 3, and aerodynamic torque was used to control yaw axis, which causes yaw stabilization. Also, we will show that tail stabilizer is passive proportional differential (PD) controller that prevents boom from inverting and provides yaw controller.

DESIGN

In this section, at first we will obtain stable and instable regions for a satellite with the use of gravity gradient boom and will design moments of inertia values to achieve stabilization. Then, with the addition of a vertical tail stabilizer, perpendicular to the boom axis, aerodynamic restoring torque can be provided. This stabilization is a differential feedback loop to provide proportional-differential controller. Here, to achieve the design goals, we should obtain suitable control gain to satisfy stability conditions and oscillations damping. The proposed method in this paper, prevents the satellite from boom inverted status in addition to oscillations damping about yaw axis, and in this way the interaction effect on the roll axis is taken care of.

– STABILITY CONDITIONS WITH ONLY GRAVITY GRADIENT

In this section, we assume that disturbance torques are zero. Satellite linearized equations of motion with gravity gradient torque can be used from Eq. 16. Therefore, stability conditions for gravity gradient satellite could be derived as bellow:

$$Iyy > Ixx > Izz \quad \& \quad Iyy < Ixx + Izz \tag{17}$$

These inequalities may be translated to the $\sigma_x - \sigma_z$ plane as it is shown as the four regions, labeled I to IV, in Figure 4 [11]. According to Figure 4, Sub-region A (Lagronge Region) in region I is normally used in practical designs of gravity gradient stabilized spacecraft and sub-region B (Debra-Delp Region) because of structural difficulties is not statically stable [2,10].

-PASSIVE PROPORTIONAL-DIFFERENTIAL CONTROLLER WITH AERODYNAMIC METHOD

Since, there is no damping factor in Eq. 16, we need to design a controller for damping oscillations as there is a need to increase response's speed. To achieve these goals, here we will design a passive PD controller. To implement this controller, we use tail stabilizer perpendicular to yaw axis that provides aerodynamic stability (Figure 5). The differentiation of deflection angle of the stabilizer is used for damping oscillations and to stabilize the satellite as follows:

(18)

The aerodynamic torque acting on the satellite due to the presence of the vertical tail stabilizer is derived from equations given below [7,8,10]:

$$T_{ad} = \frac{1}{2} -\rho V_R^2 Al \, (C_p \cos\delta_3 = \frac{\sin q_3 - \sin \; \theta_3 + \delta_3)}{\cos(q_3 + d_3)} C\tau) \tag{19}$$

$$C_p = (2 - \sigma') \, (\frac{2}{\sqrt{\pi}S} \sin(\theta_3 + \delta_3) \, x \, exp(-S^2 \sin^2(\theta_3 + \delta_3)) + (2\sin^2(\theta_3 + \delta_3) + \frac{1}{S^2}) \, erf(S \sin(\theta_3 + \delta_3)) \tag{20}$$

$$C_\tau = \frac{2\sigma}{\sqrt{\pi}S} \cos \; \theta_3 + \delta_3) \, x \; exp(-S^2 \sin^2(\theta_3 + \delta_3)) + \sqrt{\pi}S \; \sin(\theta_3 + \delta_3) \, x \, erf(S \sin(\theta_3 + \delta_3)) \tag{21}$$

All parameters mentioned in Eqs. 19-21 have been defined in appendix A. Characteristic equations of motion for roll-yaw, with yaw stabilizer and feedback loop, is obtained as:

$$s^4 + k_2 k_3 s^2 + \omega_o^2 (3\sigma_x + \sigma_x \sigma_z + 1) s^2 + 4k_2 k_3 \omega_o^2 \sigma_x s + 4\omega_o^2 \sigma_x (\omega_o^2 \sigma_z + k_2) = 0 \tag{22}$$

where k_2 is aerodynamic torque constant coefficient, that has been used for simplicity, and has no effect on stability conditions. So, as it has been shown, by designing the control law (18), we have added damping factors to the system (i.e: s^1 and s^3 coefficients). Now, using Ruth-Horowitz stability criterion we can show that this system is stable for all positive values of k_3.

NUMERICAL COMPUTATION AND SIMULATION RESULTS

- SATELLITE ATTITUDE WITHOUT EXTERNAL TORQUES

To investigate the attitude motion of the satellite without the existence of external torques, we have to solve Eqs. 9, 10 with $T_x=T_y=T_z=0$. The essential parameters of 50kg microsatellite function on circular orbit and 500km altitude are listed in appendix A. The initial condition of roll, pitch and yaw angles and their rates are assumed to be:

$$\begin{aligned} \theta_1(0) = \theta_2(0) = \theta_3(0) = 10 \text{ deg} \\ \dot{\theta}_1(0) = \dot{\theta}_2(0) = \dot{\theta}_3(0) = 10 \text{ deg / sec} \end{aligned} \quad (23)$$

The time history of the attitude motion $\theta_i(t)$ is shown in Figure 6. The results indicate that the attitude motion without the external control torques is highly oscillatory. When we apply gravity gradient torque only, the responses are shown in Figure 7, the roll and pitch axes will converge, but the yaw axis will still experience oscillation.

SATELLITE ATTITUDE WITH GRAVITY GRADIENT AND AERODYNAMIC TORQUES

Here we analyze the attitude motion of the satellite with the existence of external torques:

$$T_x = T_{ggx} \quad ; \quad T_y = T_{ggy} \quad ; \quad T_z = T_{ggz} + T_{ad} \quad (24)$$

From Eqs. 9 and 10 with the given gain value $k_3 = 17.3$, that was found empirically and using simulation results, the numerical computational results have been verified by simulation as shown in Figure 8. In this case, with the use of designed control law, the initial attitude motions are converged below the desired range.

CONCLUSIONS

The effectiveness of the attitude control method, including a gravity gradient boom and an aerodynamic stabilizer, in the attitude motion stabilization of a LEO small satellite has been investigated. Under the assumption of neglecting all disturbance torques, we only considered the control torques – gravity gradient torque and aerodynamic torque.

In this paper, we described spacecraft dynamic equations under the action of both gravity gradient torque and aerodynamic torque. Then, we obtained stable and instable regions for a satellite, with only gravity gradient boom and moments of inertia constraints. It has been shown that the system is not capable of generating torque around the yaw axis. So, we designed a proportional-differential controller by adding a vertical tail stabilizer as a simple differential feedback control loop.

Using simulation results, we indicated that if external torque did not exist, the roll, pitch and yaw angles oscillated under the initial conditions. When the gravity gradient torque was added, the roll and pitch angles ware gradually converged to zero, but the yaw motion still had an oscillation. Finally, by adding aerodynamic stabilizer, yaw motion was damped. In other words, the three-axis stabilization of the microsatellite was obtained. The paper demonestrated the feasibility of passive gg-aero stabilizatoin and provided guidelines for the satallite design to obtain the best attitude performance without sacrificing satellite lifetime.

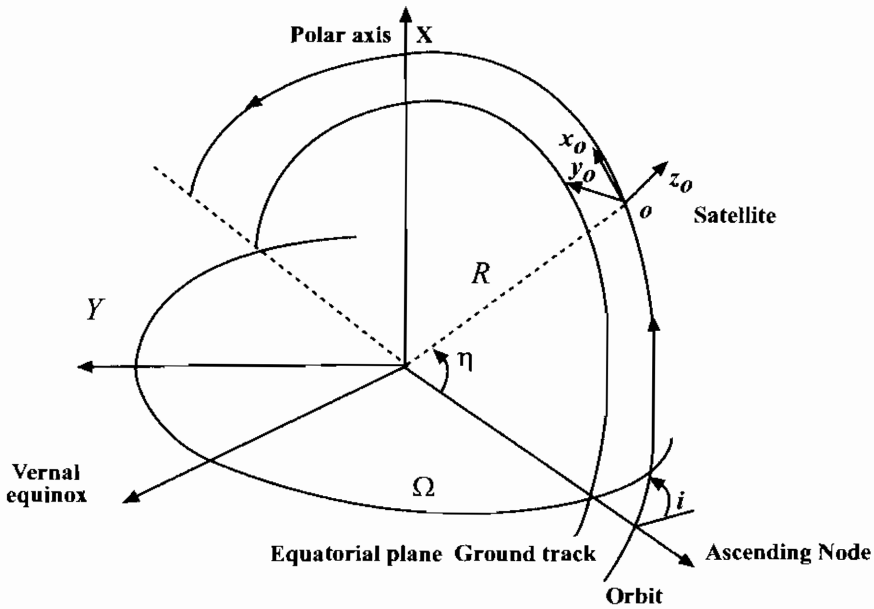

Figure 1: Inertial and orbital coordinates.

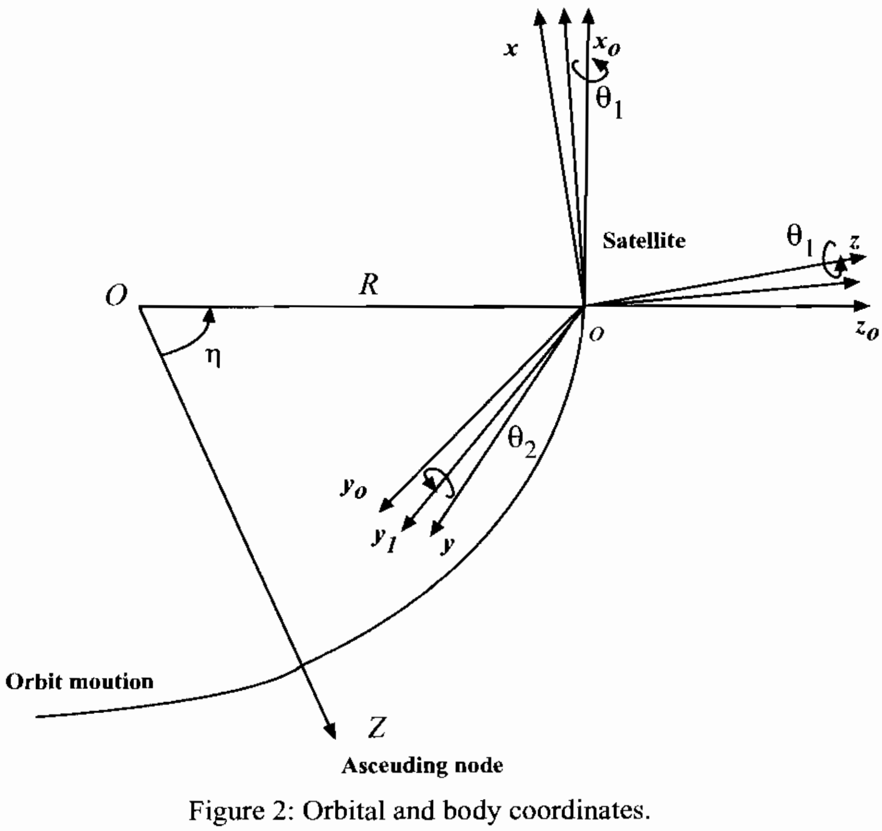

Figure 2: Orbital and body coordinates.

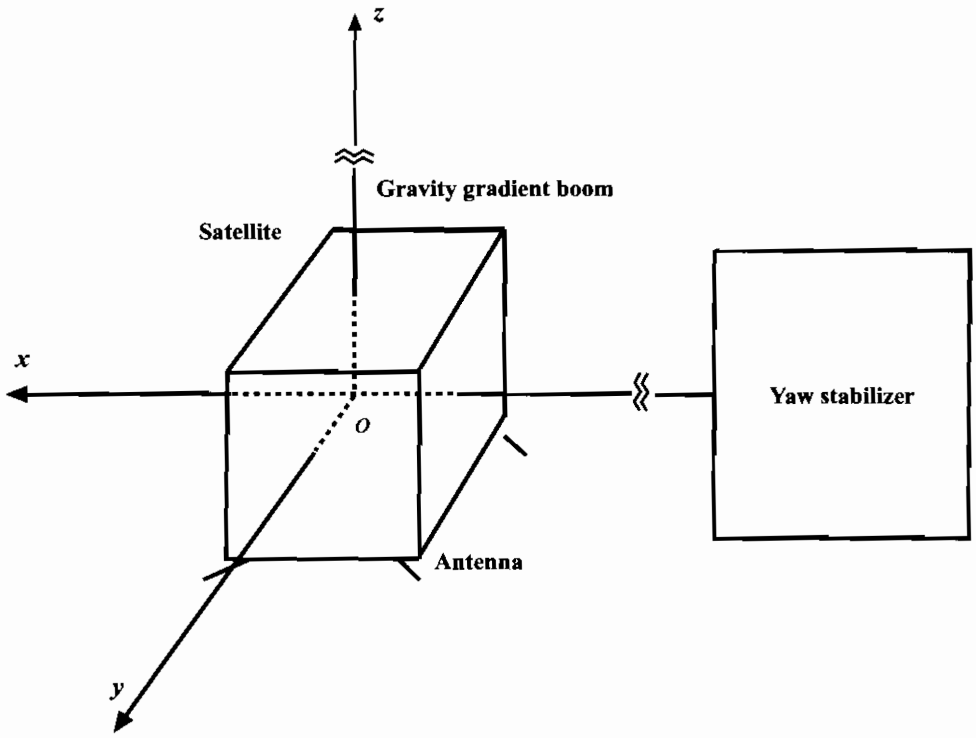

Figure 3: Satellite with gravity gradient boom and yaw stabilizer.

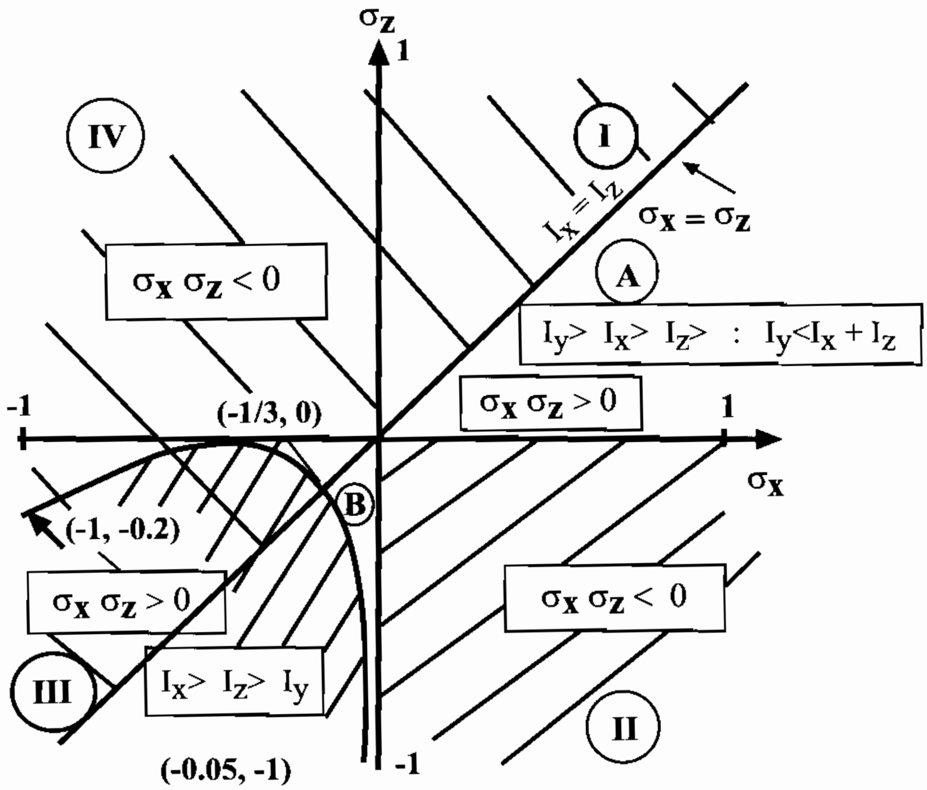

Figure 4: Stability regions for gravity gradient stabilized satellite.

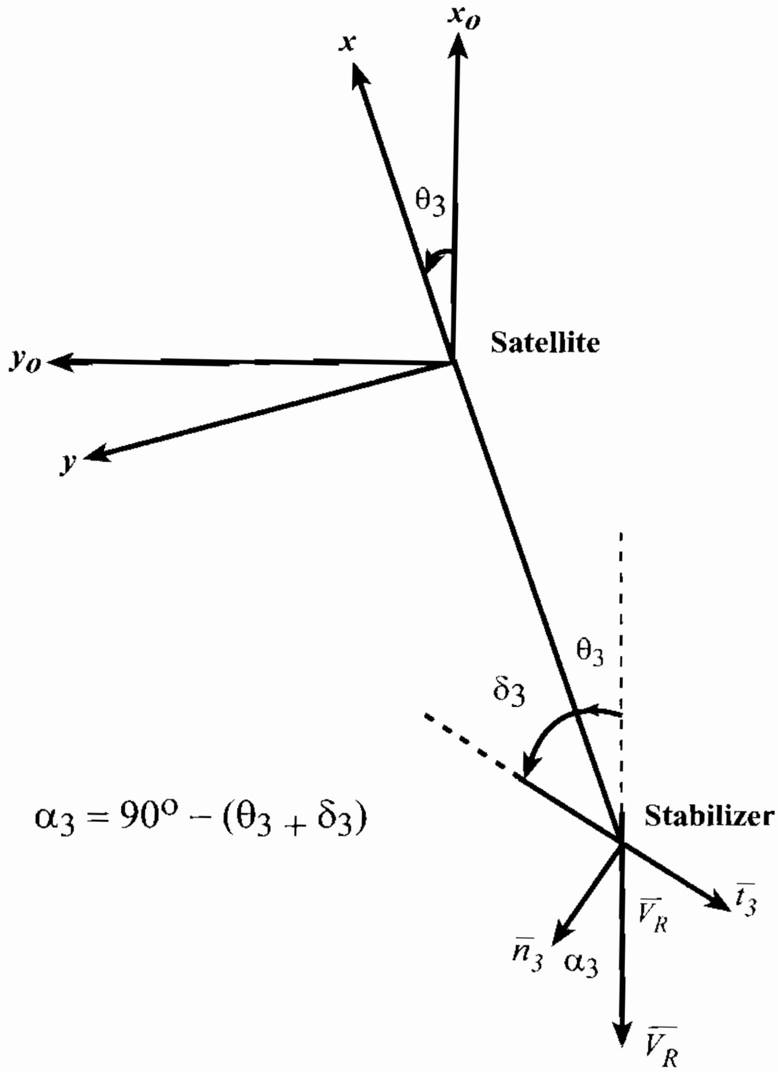

Figure 5: Geometry of yawing motion.

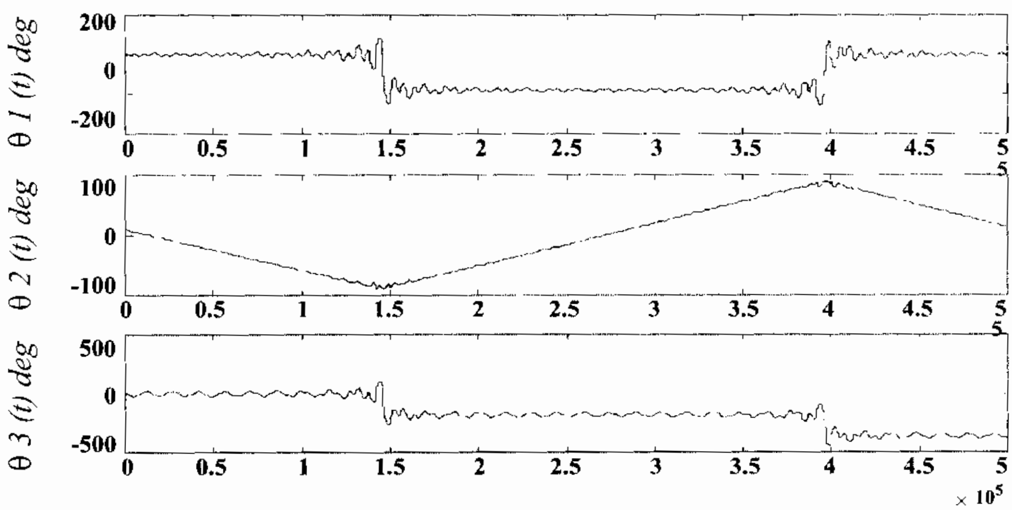

Figure 6: The time history of attitude motion without external torque.

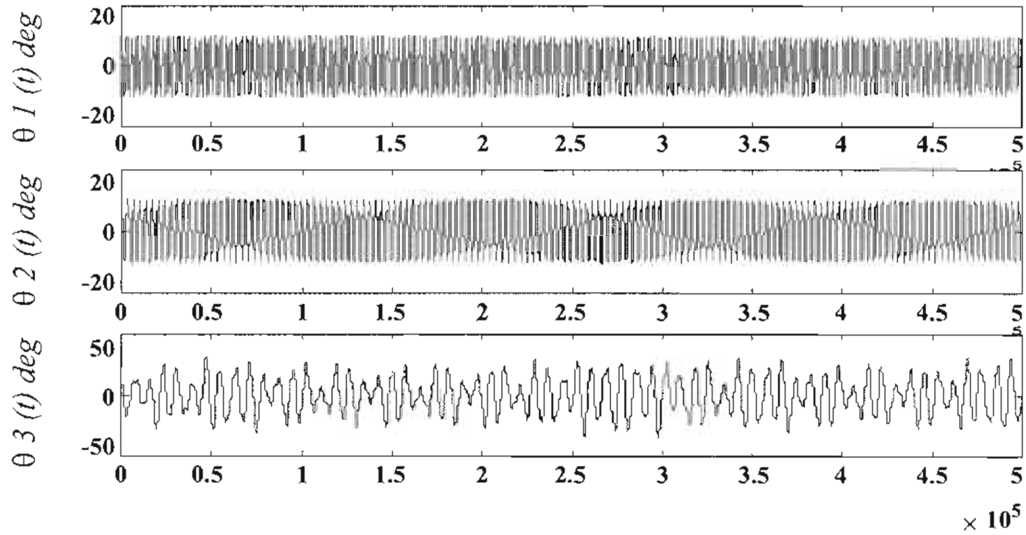

Figure 7: The time history of attitude motion with gravity gradient torque.

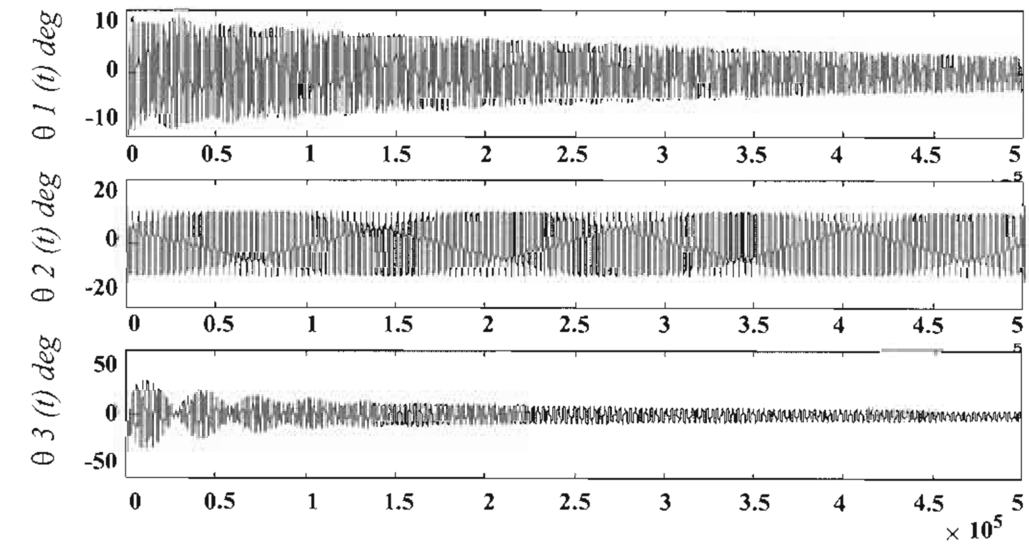

Figure 8: The time history of attitude motion with gravity gradient and aerodynamic torques.

Appendix A

Nomenclature	Description
A	area of yaw stabilizer
c_p, c	normal and shear force coefficients for yaw stabilizer
T_{gg}	gravity gradient (or gravitational) torque
T_{ad}	aerodynamic torque
$I = diag\ (I_{xx}, I_{yy}, I_{zz})$	inertia matrix for satellite
I_{xx}, I_{yy}, I_{zz}	principal moments of inertia
i	orbit inclination
R	orbital radius
L	transformation matrix
l	moment arm of yaw stabilizer
k_3	gain in yaw channel feedback loop
S	molecular speed ratio
$\vec{V}_R$	air velocity vector with respect to satellite
$V_R = \vec{V}_R / \lVert \vec{V}_R \rVert$	air velocity normal vector
$\dot{\eta}$	orbit true anomaly
η	orbital rate (ω_O)
$\theta_1, \theta_2, \theta_3$	roll, pitch, and yaw angles
δ_1	deflection angle of yaw stabilizer
ρ	atmospheric density
σ, σ'	tangential and normal accommodation coefficients
ω	angular velocity of satellite with respect to inertial
$OXYZ$	inertial coordinates
$oxyz$	principal body coordinates
$ox_oy_oz_o$	orbital coordinates

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