MULTIOBJECTIVE GEOMETRIC PROGRAMMING WITH FUZZY PARAMETERS

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Abstract - Generally, an engineering design problem has multiple objective functions. Some of these problems can be formulated as multiobjective geometric programming models. On the other hand,often in the real world, coefficients of the objective functions are not known precisely. Coefficients may be interpreted as fuzzy numbers, which lead to a multiobjective geometric programming with fuzzy parameters. In this paper, we solve the multiobjective geometric programming problem with fuzzy parameters by applying a linear ranking function. The linear ranking function is used to compare fuzzy numbers. Finally, a numerical example is given.

Keywords - Multiobjective Geometric Programming Problem, Pareto Optimality, Fuzzy Number, Ranking Function.

INTRODUCTION

The last years have witnessed a considerable interest in information fusion process, related to the improved sophistication of information and communication sciences and their tremendous social impact. In particular, a mathematical study of fuzzy optimization problem has been systematically carried out under the domination of fuzzy set theory. For a primer on fuzzy optimization models and methods and the recent developments one may refer to [6,16,17,21]. One of the most important branches of optimization is geometric programming GP. Geometric programming is an optimization technique developed to solve a class of nonlinear optimization problems especially found in engineering design and manufacturing.

The theory of GP first emerged in 1961 by Duffin and Zener. They discovered that many engineering design optimization problems have an objective function consisting of a sum of component costs. Sometimes, the objective function can be minimized almost by inspection under certain constraints where each term of the constraints are in the form of posynomial. The first publication on GP was published by Duffin and Zener in 1967 [13]. To solve algebric nonlinear programming problems subjected to linear or nonlinear constraints, several extensions proposed by different authors [2,3,4,7,8,9,10,15,18,29]. Generally, an engineering design problem has multiple objective functions. To extend geometric programming problem, Cao Bing-Yuan [7,9] first transforms it to a fuzzy state problem including the advancement of its fuzzy geometric programming model. Moreover, Cao Bing-yuan considers the situation where the coefficients are fuzzy [8,10]. Another approach which use the theory of fuzzy set to solve multiobjective GP problems is introduced by Verma [28]. Fuzzy programming is a useful method to solve multiobjective GP problems. Biswal [3] developed fuzzy programming with nonlinear membership functions approach to multiobjective GP problems. Bit [4] developed fuzzy programming with hyperbolic membership functions to solve GP with several objective functions. In this paper, which is a more completed version of [19], we first extended the pareto optimality concepts. Then the weighting method and the constraint method are used to obtain a solution for multiobjective GP problems with fuzzy parameters.

PRELIMINARIES

- RANKING FUNCTIONS

Many ranking methods can be found in fuzzy literature. Bass and Kwakernaak [1] are among the pioneers in this area. Bortolan and Degani [5] have reviewed different ranking methods. According to Chen and Hwang [11] the methods are categorized into four different groups. To examine the methods of each group see [11]. In spite of the existence of a variety of methods, no one can rank fuzzy numbers satisfactorily in all cases and situations [22].

Now, let $\tilde{a}$ be a fuzzy number, i.e. a convex normalized fuzzy subset of the real line $\mathbb{R}$ whose membership function is piecewise continuous. In this paper, we denote the set of all fuzzy numbers by $F(\mathbb{R})$. There are two important topics in the real world applications of the fuzzy set theory: arithmetic operations on fuzzy numbers and comparison of fuzzy numbers, which usually follow arithmetic operations. For arithmetic operations on the fuzzy numbers, we apply the extension principle proposed by L. A. Zadeh. Extension principle is used by many authors [25]. However, there is no common approach for comparing fuzzy numbers. Indeed, there are different approaches for ranking fuzzy numbers. A simple method for ordering the elements of $F(\mathbb{R})$ is to define a ranking function $R: F(\mathbb{R}) \to \mathbb{R}$ which maps fuzzy numbers to the real line, where a natural order exists. We then define the fuzzy number ordering as follows:

$$\tilde{a} \underset{R}{\geq} \tilde{b} \quad \text{iff} \quad R(\tilde{a}) \geq R(\tilde{b}) \tag{1}$$

$$\tilde{a} \underset{R}{>} \tilde{b} \quad \text{iff} \quad R(\tilde{a}) > R(\tilde{b}) \tag{2}$$

$$\tilde{a} \underset{R}{=} \tilde{b} \quad \text{iff} \quad R(\tilde{a}) = R(\tilde{b}) \tag{3}$$

$$\tilde{a} \underset{R}{\neq} \tilde{b} \quad \text{iff} \quad R(\tilde{a}) \neq R(\tilde{b}) \tag{4}$$

where $\tilde{a}$ and $\tilde{b}$ belong to $F(\mathbb{R})$. Also, $\tilde{a} \underset{R}{\leq} \tilde{b}$ if and only if $\tilde{b} \underset{R}{\geq} \tilde{a}$.
It is obvious that there can be more than one ranking function.
Example 2.1. Roubens ranking function is defined as [14]:

$$R(\tilde{a}) = \frac{1}{2}\int_0^1 (\inf \tilde{a}_\alpha + \sup \tilde{a}_\alpha)\, d_\alpha. \tag{5}$$

where $\tilde{a}_\alpha$ is an α-level set of $\tilde{a}$.
Let $\tilde{a} = (a^L, a^U, \alpha, \beta)$ be a trapezoidal fuzzy number, defined by:

$$\tilde{a}(x) = \begin{cases} 1 & \text{if } a^L \leq x \leq a^u, \\ 1 - \frac{a^L - x}{\alpha} & \text{if } a^L - \alpha \leq x \leq a^L, \\ 1 - \frac{x - a^U}{\beta} & \text{if } a^U < x \leq a^U + \beta, \\ 0 & \text{Otherwise.} \end{cases}$$

According to the above definition, it is easy to show that $R(\tilde{a}) = \frac{1}{2}(a^L + a^U + \frac{1}{2}(\beta - \alpha))$ [14].

Definition 2.1. By a linear ranking function we mean a ranking function $R: F(\mathbb{R}) \rightarrow \mathbb{R}$ such that $R(k\tilde{a} + \tilde{b}) = kR(\tilde{a}) + R(\tilde{b})$, $\tilde{a}, \tilde{b} \in F(\mathbb{R})$ and $k \in \mathbb{R}$. It is easy to show that Roubens ranking function is linear [14].

Remark 2.1. In the remainder of this paper we assume that R is a linear ranking function.

By a fuzzy function, $f(., \tilde{a}) : \mathbb{R}^n \rightarrow F(\mathbb{R})$, we mean an ordinary function from $\mathbb{R}^n$ to $F(\mathbb{R})$, with a vector of fuzzy numbers, $\tilde{a} = (\tilde{a}_1, \ldots, \tilde{a}_p)$, as parameters. Specially by a polynomial fuzzy function, we mean an ordinary polynomial function with fuzzy numbers as parameters, for example: $f(x, \tilde{a}) = \sum_{k=1}^{p} \tilde{a}_k \prod_{n=1}^{N} x_n^{kn}$.

Definition 2.2. Let R be a linear ranking function. A fuzzy function $f(x, \tilde{a})$ defined on a nonempty convex set S in R^n is said to be

a) R-convex if

$$f(\lambda x_1 + (1-\lambda)x_2, \tilde{a}) \underset{R}{\leq} \lambda f(x_1, \tilde{a}) + (1-\lambda) f(x_2, \tilde{a})$$
$$\forall x_1, x_2 \in S \text{ and } \forall \lambda \in [0, 1].$$

b) R-concave if

$$f(\lambda x_1 + (1-\lambda)x_2, \tilde{a}) \underset{R}{\geq} \lambda f(x_1, \tilde{a}) + (1-\lambda) f(x_2, \tilde{a})$$
$$\forall x_1, x_2 \in S \text{ and } \forall \lambda \in [0, 1].$$

- NOTATIONS

In this section, some notations are introduced which are used in the remainder of this paper.

Let $x = (x_1, x_2, \ldots, x_k)'$, $y = (y_1, y_2, \ldots, y_k)'$, $\in \mathbb{R}^k$ and $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_k)'$,

$\tilde{y} = (\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_k)', \in (F(\mathbf{R}))^k$, then:

$x \geq y$ iff $x_i \geq yi$, $j = 1, 2, \ldots, k$,

$x > y$ iff $x \geq y$, & $x \neq y$,

$x >> y$ iff $x_i > yi$, $j = 1, 2, \ldots, k$,

$\tilde{x} \underset{R}{\geq} \tilde{y}$ iff $\tilde{x}_i \underset{R}{\geq} \tilde{y}i$, $j = 1, 2, \ldots, k$,

$\tilde{x} \underset{R}{>} \tilde{y}$ iff $\tilde{x} \underset{R}{\geq} \tilde{y}$, & $\tilde{x} \underset{R}{\neq} \tilde{y}$,

$\tilde{x} \underset{R}{>>} \tilde{y}$ iff $\tilde{x}_i \underset{R}{>} \tilde{y}i$. $j = 1, 2, \ldots, k$,

Moreover, a function h : $(F(\mathbf{R}))^k \to F(\mathbf{R})$ is called:

- R-monotone increasing iff:

$$\tilde{x} \underset{R}{>} \tilde{y} \Rightarrow h(\tilde{x}) \underset{R}{\geq} h(\tilde{y}),$$

- R-strong monotone increasing iff:

$$\tilde{x} \underset{R}{>} \tilde{y} \Rightarrow h(\tilde{x}) \underset{R}{>} h(\tilde{y}),$$

- R-strict monotone increasing iff:

$$\tilde{x} \underset{R}{>>} \tilde{y} \Rightarrow h(\tilde{x}) \underset{R}{>} h(\tilde{y}).$$

We shall say that the real number t corresponds to the fuzzy number $\tilde{t}$ if $t = R(\tilde{t})$.

GP AND MULTIOBJECTIVE GP WITH FUZZY PARAMETERS

- GP WITH FUZZY PARAMETERS

In the following, we define the geometric programming (GP) problem with fuzzy parameters and propose a method for solving it.

Definition 3.1. Let $f(x, \tilde{a})$ and $g_i(x, \tilde{b}_i)$, $i = 1, 2, \ldots, m$, be polynomial fuzzy functions, where $\tilde{a} = (\tilde{a}_1, \ldots, \tilde{a}_p)$ and $\tilde{b}_i = (\tilde{b}_{i_1}, \ldots, \tilde{b}_{i_p})$ represent vector of fuzzy numbers involved in $f(x, \tilde{a})$ and $g_i(x, \tilde{b}_i)$ respectively. Then, the model

$$\begin{aligned} &\max: \tilde{z} \underset{R}{=} f(x, \tilde{a}) \\ &\text{s.t. } g_i(x, \tilde{b}_i) \leq 0, \qquad i = 1, 2, \ldots m, \\ &\qquad x > 0, \end{aligned} \tag{6}$$

is called a geometric programming with fuzzy parameters (GPF) problem.

Definition 3.2. Any set of xj which satisfies the set of constraints of GPF is called a solution for GPF. Let Q be the set of all solutions of GPF. We shall say that $x^* \in Q$ is an optimal solution for GPF if $f(x, \tilde{a}) \underset{R}{\leq} f(x^*, \tilde{a})$ for all $x \in Q$.

The following theorem is similar to Lemma 3.1. of [20] for solving fuzzy number linear programming problems. Here, we use it to reduce any GPF to a GP problem in the classical form.

Theorem 3.1. The following problem and GPF are equivalent:

$$\begin{aligned} \max:\ & z = f(\mathrm{x}, \mathrm{a}), \\ \text{s.t. } & g_i(\mathrm{x}, b_i) \le 0, \qquad i = 1, 2, \dots \mathrm{m}, \\ & \mathrm{x} > 0, \end{aligned} \tag{7}$$

where $\mathrm{a} = (a_1, \dots, a_p)$ and $\mathrm{b}_i = (b_{i_1}, \dots, b_{p_i})$, are vectors of real numbers corresponding to the fuzzy vectors $\tilde{\mathrm{a}} = (\tilde{a}_1, \dots, \tilde{a}_p)$ and $\tilde{b}_i = (\tilde{b}_i, \dots, \tilde{b}_{i_p})$, respectively.

Proof: It is straightforward, by using linearity of ranking function.

- MULTIOBJECTIVE GP WITH FUZZY PARAMETERS

In this section, we consider a multiobjective GP problem in which all decision parameters are fuzzy numbers and investigate some methods to solve it.

Definition 3.3. The model

$$\begin{aligned} \max:\ & \tilde{z} \underset{R}{=} (f_1(\mathrm{x}, \tilde{\mathrm{a}}_1), f_2(\mathrm{x}, \tilde{\mathrm{a}}_2), \dots, f_k(\mathrm{x}, \tilde{\mathrm{a}}_k)), \\ & \text{s.t. } g_i(\mathrm{x}, \tilde{b}_i) \underset{R}{\le} 0, \quad i = 1, 2, \dots \mathrm{m}, \\ & \qquad\qquad \mathrm{x} > 0, \end{aligned} \tag{8}$$

where $\mathrm{x} \in \mathbb{R}^n$, $f_1(\mathrm{x}, \tilde{\mathrm{a}}_1)$, $f_2(\mathrm{x}, \tilde{\mathrm{a}}_2)$, ..., $f_k(\mathrm{x}, \tilde{\mathrm{a}}_k)$ and $g_1(\mathrm{x}, \tilde{b}_1)$, $g_2(\mathrm{x}, \tilde{b}_2)$, ..., $g_m(\mathrm{x}, \tilde{b}_m)$ are fuzzy polynomial functions from $\mathbb{R}^n$ to $F(\mathbb{R})$, is called a *multiobjective geometric programming with fuzzy parameters* (MGPF) problem.

It is noted that the objectives are non-commensurable and inherently conflicting in nature in a multiobjective decision making environment.

Set $\mathrm{X} = \{\mathrm{x} \in \mathbb{R}^n \mid gi(\mathrm{x}, \tilde{b}_i) \underset{R}{\le} 0,\ i = 1, 2, \dots, m, \mathrm{x} > 0\}$. **X** is the feasible solution set for MGPF. Since there are fuzzy numbers as parameters in the objective functions of (8), we extend the concept of pareto optimality in classical multiobjective GP as follows.

Definition 3.4. $\mathrm{x}^* \in \mathrm{X}$ is an R-pareto optimal solution for (8) if and only if there does not exist $\mathrm{x} \in \mathrm{X}$ such that $(f_i(\mathrm{x}, \tilde{\mathrm{a}}_i), \underset{R}{\ge} f_i(\mathrm{x}^*, \tilde{\mathrm{a}}_i)$ for all i and $f_i(\mathrm{x}, \tilde{\mathrm{a}}_i) \underset{R}{\ne} f_j(\mathrm{x}^*, \tilde{\mathrm{a}}_j)$ for at least one j.

A relaxed definition of R-pareto optimality is as follows:

Definition 3.5. $\mathrm{x}^* \in \mathrm{X}$ is a weak R-pareto optimal solution for (8) if and only if there does not exist $\mathrm{x} \in \mathrm{X}$ such that $f_i(\mathrm{x}, \tilde{\mathrm{a}}_i), \underset{R}{>} f_i(\mathrm{x}^*, \tilde{\mathrm{a}}_i)$ for all i.

Theorem 3.2. Let $h:(F(\mathbb{R}))^k \to F(\mathbb{R})$ and x^* be an optimal solution for

$$\begin{aligned} \max:\ & \tilde{z} \underset{R}{=} h(f_1(\mathrm{x}, \tilde{\mathrm{a}}_1), f_2(\mathrm{x}, \tilde{\mathrm{a}}_2), \dots, f_k(\mathrm{x}, \tilde{\mathrm{a}}_k)), \\ & \qquad \text{s.t.} \quad \mathrm{x} \in \mathrm{X}, \end{aligned} \tag{9}$$

then:

a) if h is R-strict monotone increasing on $F(\mathbb{R}))^k$, then x^* is a weak R-pareto optimal solution for MGPF.

b) if h is R-strong monotone increasing on $(F(\mathbb{R}))^k$, then x* is an R-pareto optimal solution for MGPF.

c) if h is R-monotone increasing on $(F(\mathbb{R}))^k$, and x* is the unique optimal solution to (9), then x* is an R-pareto optimal solution for MGPF.

Proof:

- For case (a), suppose that x* is not a weak R-pareto optimal solution for (9). Then, there exists $x \in X$ such that $f_i(x, \tilde{a}_i), \underset{R}{>} f_i(x^*, \tilde{a}_i)$ for all i. So we have

$$(f_1(x, \tilde{a}_1), f_2(x, \tilde{a}_2), \ldots, f_k(x, \tilde{a}_k)) \underset{R}{>>} (f_1(x^*, \tilde{a}_1), f_2(x^*, \tilde{a}_2), \ldots, f_k(x^*, \tilde{a}_k)),$$

Now since h is R-strict monotone increasing on $F(\mathbb{R}))^k$, we have

$$h(f_1(x, \tilde{a}_1), f_2(x, \tilde{a}_2), \ldots, f_k(x, \tilde{a}_k)) \underset{R}{>} h(f_1(x^*, \tilde{a}_1), f_2(x^*, \tilde{a}_2), \ldots, f_k(x^*, \tilde{a}_k))$$

This contradicts the optimality of x*.

In supporting our proof, we suppose that x* is not an R-pareto optimal solution for (9). Then, there exists $x \in X$ such that

$$f_i(x, \tilde{a}_i) \underset{R}{\geq}, f_i(x^*, \tilde{a}_i) \text{ for all } i \text{ and } f_i(x, \tilde{a}_j) \underset{R}{\neq} f_i(x^*, \tilde{a}_i) \text{ for at least one j.}$$

In other words,

$$(f_1(x, \tilde{a}_1), f_2(x, \tilde{a}_2), \ldots, f_k(x, \tilde{a}_k)) \underset{R}{>} (f_1(x^*, \tilde{a}_1), f_2(x^*, \tilde{a}_2), \ldots, f_k(x^*, \tilde{a}_k)).$$

Hence:

- For case (b) suppose h is R-strong onotone increasing on $F(\mathbb{R}))^k$, then, we have

$$h(f_1(x, \tilde{a}_1), f_2(x, \tilde{a}_2), \ldots, f_k(x, \tilde{a}_k)) \underset{R}{>} h(f_1(x^*, \tilde{a}_1), f_2(x^*, \tilde{a}_2), \ldots, f_k(x^*, \tilde{a}_k)),$$

which contradicts the optimality of x* .

- For case (c), suppose h is R-monotone increasing on $(F(\mathbb{R}))^k$. Then, we have

$$h(f_1(x, \tilde{a}_1), f_2(x, \tilde{a}_2), \ldots, f_k(x, \tilde{a}_k)) \underset{R}{\geq} h(f_1(x^*, \tilde{a}_1), f_2(x^*, \tilde{a}_2), \ldots, f_k(x^*, \tilde{a}_k)),$$

which contradicts the fact that x* is the unique optimal solution to (9).

TWO APPROACHES FOR SOLVING MGPF

The well-known scalarization methods provide an approach for obtaining pareto optimal solutions to multiobjective programming problems. In this section, we apply scalarizationtype methods to characterize R-pareto optimal solutions for MGPF.

- WEIGHTING METHOD

The weighting method to obtain an R-pareto optimal solution of MGPF is to solve the weighting problem formulated by taking the weighted sum of the objective functions of the original MGPF. Thus, weighting problem for MGPF could be defined by:

$$\max: \tilde{z} \underset{R}{=} \sum_{i=1}^{k} w_i f_i (x, \tilde{a}_i), \quad \text{s.t.} \quad x \in X, \qquad (10)$$

where $w = (w_1, w_2, \ldots, w_k) \geq 0$ is the vector of weighting coefficients assigned to the objective functions. The relationship between the optimal solutions of the weighting problem (10) and R-pareto optimal solutions of MGPF can be characterized by the following theorems.

Theorem 4.1. Let x^* be an optimal solution of the weighting problem (10) for some $w - 0$.

a) If $w > 0$, then x^* is a weak R-pareto optimal solution for MGPF.

b) If $w >> 0$, then x^* is an R-pareto optimal solution for MGPF.

c) If x^* is a unique optimal solution for (10) and $w > 0$, then x^* is an R-pareto optimal solution for MGPF.

Proof: If we set $h(f_1(x, \tilde{a}_1), f_2(x, \tilde{a}_2), \ldots, f_k(x, \tilde{a}_k)) = \sum_{i=1}^{k} w_i f_i (x, \tilde{a}_i)$, then proof may be directly derived from Theorem 4.2. □

Theorem 4.2. Assume that R is a linear ranking function and all fuzzy function f_i $(x, \tilde{a}_i)$ are R-concave and all fuzzy functions $g_i (x, \tilde{b}_i)$ are R-convex, in the MGPF. If x^* is an R-pareto optimal solution of the MGPF, then there exists $w > 0$ such that x^* is an optimal solution for (10).

Proof: The proof is straightforward, if we use the linearity of ranking function R and Theorem 4.9 of [25]. □

By Theorem 4.1, among the other results, each $x \in X$ which is an optimal solution of the weighting problem (10) for any $w >> 0$ is an R-pareto optimal solution for MGPF.

On the other hand, under the conditions of Theorem 3.3, at least one $w > 0$ is associated with every R-pareto optimal solution, that causes the weighting problem (10) to have it as an optimal solution.

Note that different methods exist to help the decision maker to assign weights to the objective functions [12,23,24,26].

- CONSTRAINT METHOD

The constraint method for characterizing R-pareto optimal solutions is to solve the following constrainted programing problem, which formulated by taking one objective function as the objective function and letting the others to set inequality constraints.

$$\max: \tilde{z} \underset{R}{=} f_i (x, \tilde{a}_i), \quad \text{s.t.} \quad f_i (x, \tilde{a}_j) - \tilde{\varepsilon}_j, \quad j = 1, 2, \ldots, k \quad j \neq i, \quad x \in X, \qquad (11)$$

where $\tilde{\varepsilon}_j$ is the minimum acceptable fuzzy values for objectives corresponding to $j \neq i$.

Theorem 4.3. Let x* be an optimal solution of the constrainted problem (11) for some $\tilde{\varepsilon}_j$, $j = 1, 2, \ldots, k$ and $j \neq i$.

a) x* is a weak R-pareto optimal solution for MGPF.

b) If x* is a unique optimal solution for (11), then x* is an R-pareto optimal solution for MGPF.

Proof:

a) If x* is not a weak R-pareto optimal solution for MGPF, then there exists $x \in X$ such that $f_j(x, \tilde{a}_j), \underset{R}{>} f_j(x^*, \tilde{a}_j)$ for all j. Specially $f_i(x, \tilde{a}_i) \underset{R}{>} f_i(x^*, \tilde{a}_i)$, which contradicts the assumption that x* is an optimal solution for (11).

b) Let x* be a unique optimal solution for (11), which is not an R-pareto optimal solution for MGPF, then there exists $x \in X$ such that $f_j(x, \tilde{a}_j), \underset{R}{\geq} f_j(x^*, \tilde{a}_j)$ for all j and $f_l(x, \tilde{a}_l), \underset{R}{\neq} f_l(x^*, \tilde{a}_l)$, for at least one l. This means either

$$f_j(x, \tilde{a}_j) \underset{R}{\geq} f_j(x^*, \tilde{a}_j) \underset{R}{\geq} \tilde{\varepsilon}_j, \quad j = 1, 2, \ldots, k,\ j \neq i \quad f_i(x, \tilde{a}_i) \underset{R}{=} f_i(x^*, \tilde{a}_i),$$

$$f_j(x, \tilde{a}_j) \underset{R}{\geq} f_j(x^*, \tilde{a}_j) \underset{R}{\geq} \tilde{\varepsilon}_j, \quad j = 1, 2, \ldots, k,\ j \neq i \quad f_i(x, \tilde{a}_i) \underset{R}{>} f_i(x^*, \tilde{a}_i),$$

which contradicts the assumption that x* is a unique optimal solution for (11).□

Theorem 4.4. If x* is an R-pareto optimal solution of the MGPF, then x* is an optimal solution for (11) for some $\tilde{\varepsilon}_j$, $j = 1, 2, \ldots, k$ and $j \neq i$.

Proof: Suppose that x* is not an optimal solution for (11) for any $\tilde{\varepsilon}_j$, $j = 1, 2, \ldots, k$ and $j \neq i$. Then, there exists $x \in X$ such that $f_j(x, \tilde{a}_j), \underset{R}{\geq} \tilde{\varepsilon}_j \underset{R}{=} f_j(x^*, \tilde{a}_j)$, $j = 1, 2, \ldots, k$, $j \neq i$, $f_i(x, \tilde{a}_i) \underset{R}{>} f_i(x^*, \tilde{a}_i)$. This contradicts the assumption that x* is an R-pareto optimal solution of the MGPF. □

Thus, to find an R-pareto optimal solution for MGPF, we must select one of the objectives for maximization subject to: $\tilde{\varepsilon}_j$ to be lower bounds on the other objectives to form (11). Then, by Theorem 3.4, if x* is an optimal solution (unique optimal solution) for (11), then x* is a weak R-pareto (R-pareto) optimal solution for MGPF.

It is noted that a different solution is generally achieved with the change of objective function for optimizing it in the decision making context. Therefore, the noticeable task in the constraint method is selecting one objective function for optimization and the lower bounds for the other objectives. It is a reasonable task for the decision maker. He or she may determine them subjectively. Moreover, the decision maker may use the interactive nature of the constraint method. In other words, the decision maker first configure the problem to produce a candidate solution. Then, by examination of this solution, the decision maker inputs information to another configuration that will lead to a better solution. Repeating this pattern, the decision maker will sooner or later reach a point at which he or she will stop (perhaps by losing patience with the procedure). Then, from the series of solution generated, the decision maker will pick the point with which he or she feels most comfortable as the final solution [27]. For more details see section 8.5. of [27].

NUMERICAL EXAMPLE

To illustrate the use of constraint method to solve MPGF, we consider the following numerical example:

Find x_1, x_2, and x_3, so as to:

$$\begin{aligned}
\min\ &: \tilde{z}_1 \underset{R}{=} (39,41,1,1)x_1^{-1}x_2^{\frac{-1}{2}}x_3^{-1} + (18,22,2,2)x_1x_3 + (19,22,3,1)x_1x_2x_3, \\
\min\ &: \tilde{z}_2 \underset{R}{=} (39,41,1,1)x_1^{-1}x_2^{-1}x_3^{-1} + (19,21,1,1)x_1^{\frac{1}{3}}x_3^{\frac{3}{4}}, \\
s.t.\ & (0,2,1,1)x_1^{-2}x_2^{-2} + (2,6,2,2)x_2^{\frac{1}{2}}x_3^{-1} \underset{R}{\le} (\tfrac{1}{2},\tfrac{3}{2},1,1), \\
& x_1, x_2, x_3 > 0.
\end{aligned} \tag{12}$$

Now, rewrite the problem as a maximization model. If the decision maker select the first objective for maximization and add the second objective as a constraint with as its lower bound, then we have the following GPF:

$$\begin{aligned}
\min\ &: \tilde{z}_1 \underset{R}{=} (-41,-39,1,1)x_1^{-1}x_2^{\frac{-1}{2}}x_3^{-1} + (-22,-18,2,2)x_1x_3 + (-22,-19,3,1)x_1x_2x_3, \\
s.t.\ & (0,2,1,1)x_1^{-2}x_2^{-2} + (2,6,2,2)x_2^{\frac{1}{2}}x_3^{-1} \underset{R}{\le} (\tfrac{1}{2},\tfrac{3}{2},1,1), \\
& (-41,-39,2,2)x_1^{-1}x_2^{-1}x_3^{-1} + (-21,-19,1,1)x_1^{\frac{1}{3}}x_3^{\frac{3}{4}} \underset{R}{\ge} (-60,-50,6,6), \\
& x_1, x_2, x_3 > 0.
\end{aligned} \tag{13}$$

Use the Roubens ranking function and apply Theorem 2.1 to (13), then the following GP problem will obtain:

$$\begin{aligned}
\min\ &: z_1 = -40x_1^{-1}x_2^{\frac{-1}{2}}x_3^{-1} - 20x_1x_3 - 20x_1x_2x_3, \\
s.t.\ & x_1^{-2}x_2^{-2} + 4x_2^{\frac{1}{2}}x_3^{-1} \le 1, \\
& -40x_1^{-1}x_2^{-1}x_3^{-1} - 20x_1^{\frac{1}{3}}x_3^{\frac{3}{4}} \ge -55, \\
& x_1, x_2, x_3 > 0.
\end{aligned} \tag{14}$$

The optimal solution of (14) is $(x_1, x_2, x_3) = (1,1,2)$, with $z_1 = -100$, $z_2 = -53.63585$. This is an R-pareto optimal solution for MGPF (12).

CONCLUSIONS

The parameters of mathematical programming models for many real world problems may only be stated imprecisely and this leads to the formulation of mathematical programming models with fuzzy parameters. In this paper, a multiobjective geometric programming problem in which parameters are fuzzy numbers is discussed. In order to define the concept of R-pareto optimal solution for this problem, a linear ranking function for comparing fuzzy numbers is used. A numerical example is given to clarify the theorems related to the constraint method appeared in the paper.

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